The Complete Solution of the Core-Periphery Model for two Regions Using Successive Approximations

Catalin Angelo Ioan[1], **Gina Ioan**[2]

Abstract: In this paper it is give the complete solution, using the Newton's method of approximation, for the well known Krugman's Core-Periphery model for two regions. After the process of reduction of the system of conditions, using appropriate substitutions, it is obtained one equation which is the key of the problem's solving. After the presentation of the iterative formula which gives the solution, the principal indicators (regional incomes, the prices indexes of manufactured goods and the real wage of workers) are calculated.

Keywords Core-Periphery; region; wage

JEL Classification: R12

1. Introduction

The Core-Periphery Model developed by Krugman in a series of papers starting in 1991 has whip up the scientific world for the fact that they are non-analytical solvable.

In this paper, we shall give a complete solution, using the reduction of the system of conditions to one non-linear equation, which can be solved by successive approximations.

After this, we can give the principal behavior of the quantities of the model.

Let therefore ([3]) two regions with two types of production: agriculture and manufacture.

We shall suppose that the utility function is of Cobb-Douglas type, that is:

(1) $U=C_M^{\mu}C_A^{1-\mu}$

[1] Associate Professor, PhD, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, Corresponding author: catalin_angelo_ioan@univ-danubius.ro

[2] Assistant Professor, PhD in progress, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, e-mail: gina_ioan@univ-danubius.ro

where C_A and C_M are the consumptions of the agricultural goods and of manufactures aggregate respectively and $\mu \in (0,1)$ represents the share of expenditure which is received by the manufacturers.

The behavior of μ is essential in the convergence of both regions.

Supposing that, after a scaling, that the total labor is unitary, let denote with L_1 and L_2 the worker supply in the corresponding regions. Supposing that the peasants (which produce the agricultural goods) are fixed in each region, we have that:

(2) $L_1+L_2=\mu$

In what follows, we shall made some assumptions: first, we suppose that the wage rate of peasants is numeraire, second – the transportation costs of agricultural goods are null, third – we assume that for each unit of manufactured goods shipped from one region to another only a fraction $\tau<1$ arrives. Let note $T=\frac{1}{\tau}>1$.

We shall define $\lambda=\frac{L_1}{\mu}\in(0,1)$ - the share of manufacturing labor force in region 1 and, of course, $1-\lambda=\frac{L_2}{\mu}$ is the share of manufacturing labor force in region 2.

Let now the w_1 the wage rate of workers in region 1 and also w_2 the wage rate of workers in region 2.

Let denote with Y_1 and Y_2 the regional incomes, G_1, G_2 – the true prices indexes of manufactured goods for people who live in region 1, respectively in region 2, ω_1, ω_2 – the real wage of workers living in region 1, respectively 2.

Finally we note with $\sigma>1$ - the elasticity of demand.

The equations of short-run equilibrium are ([3]):

(3) $Y_1=\mu\lambda w_1+\frac{1-\mu}{2}$

(4) $Y_2=\mu(1-\lambda)w_2+\frac{1-\mu}{2}$

(5) $G_1^{1-\sigma}=\lambda w_1^{1-\sigma}+(1-\lambda)(w_2T)^{1-\sigma}$

(6) $G_2^{1-\sigma}=\lambda(w_1T)^{1-\sigma}+(1-\lambda)w_2^{1-\sigma}$

(7) $w_1^{\sigma}=Y_1G_1^{\sigma-1}+Y_2G_2^{\sigma-1}T^{1-\sigma}$

(8) $w_2^{\sigma}=Y_1G_1^{\sigma-1}T^{1-\sigma}+Y_2G_2^{\sigma-1}$

(9) $\omega_1=w_1G_1^{-\mu}$

(10) $\omega_2=w_2G_2^{-\mu}$

2. The Complete Solution

Substituting (5) and (6) in (7) and (8) we obtain:

(11) $w_1^{\sigma}[\lambda^2 w_1^{2-2\sigma}T^{1-\sigma}+\lambda(1-\lambda)w_1^{1-\sigma}w_2^{1-\sigma}+\lambda(1-\lambda)\, w_1^{1-\sigma}w_2^{1-\sigma}T^{2-2\sigma}+(1-\lambda)^2w_2^{2-2\sigma}T^{1-\sigma}]=$
$\lambda Y_1T^{1-\sigma}w_1^{1-\sigma}+(1-\lambda)Y_1w_2^{1-\sigma}+\lambda Y_2w_1^{1-\sigma}T^{1-\sigma}+(1-\lambda)Y_2T^{2-2\sigma}w_2^{1-\sigma}$

(12) $w_2^{\sigma}[\lambda^2 w_1^{2-2\sigma}T^{1-\sigma}+\lambda(1-\lambda)w_1^{1-\sigma}w_2^{1-\sigma}+\lambda(1-\lambda)w_1^{1-\sigma}w_2^{1-\sigma}T^{2-2\sigma}+(1-\lambda)^2w_2^{2-2\sigma}T^{1-\sigma}]=$
$\lambda Y_1w_1^{1-\sigma}T^{2-2\sigma}+(1-\lambda)Y_1w_2^{1-\sigma}T^{1-\sigma}+\lambda Y_2w_1^{1-\sigma}+(1-\lambda)Y_2T^{1-\sigma}w_2^{1-\sigma}$.

Replacing (3), (4) in (11), (12) we have (after simplifications):

(13) $\lambda^2w_1^{2-\sigma}T^{1-\sigma}(1-\mu)+\lambda(1-\lambda)w_1w_2^{1-\sigma}[1+T^{2-2\sigma}-\mu]-2\lambda\frac{1-\mu}{2}w_1^{1-\sigma}T^{1-\sigma}+$
$(1-\lambda)^2w_1^{\sigma}w_2^{2-2\sigma}T^{1-\sigma}-(1-\lambda)\frac{1-\mu}{2}w_2^{1-\sigma}[1+T^{2-2\sigma}]-\mu\lambda(1-\lambda)w_1^{1-\sigma}w_2T^{1-\sigma}-\mu(1-\lambda)^2w_2^{2-\sigma}T^{2-2\sigma}=0$

(14) $-\mu\lambda^2w_1^{2-\sigma}T^{2-2\sigma}-\mu\lambda(1-\lambda)w_1w_2^{1-\sigma}T^{1-\sigma}-\lambda\frac{1-\mu}{2}w_1^{1-\sigma}[1+T^{2-2\sigma}]+$
$+(1-\lambda)^2w_2^{2-\sigma}T^{1-\sigma}[1-\mu]+\lambda(1-\lambda)w_1^{1-\sigma}w_2[1+T^{2-2\sigma}-\mu]+\lambda^2w_1^{2-2\sigma}w_2^{\sigma}T^{1-\sigma}-$
$-2(1-\lambda)\frac{1-\mu}{2}w_2^{1-\sigma}T^{1-\sigma}=0.$

Let note now: $X=\frac{w_1}{w_2}$. After a dividing with $w_2^{2-\sigma}$ in (13), (14) we have:

(15) $\lambda^2(1-\mu)T^{1-\sigma}X^{2-\sigma}+\lambda(1-\lambda)(1+T^{2-2\sigma}-\mu)X+(1-\lambda)^2X^{\sigma}T^{1-\sigma}-\mu\lambda(1-\lambda)X^{1-\sigma}T^{1-\sigma}-\mu(1-\lambda)^2T^{2-2\sigma}-\frac{1-\mu}{2}\frac{1}{w_2}[2\lambda T^{1-\sigma}X^{1-\sigma}+(1-\lambda)(1+T^{2-2\sigma})]=0$

(16) $-\mu\lambda^2T^{2-2\sigma}X^{2-\sigma}-\mu\lambda(1-\lambda)T^{1-\sigma}X+(1-\lambda)^2(1-\mu)T^{1-\sigma}+\lambda(1-\lambda)(1-\mu+T^{2-2\sigma})X^{1-\sigma}+\lambda^2T^{1-\sigma}X^{2-2\sigma}-\frac{1-\mu}{2}\frac{1}{w_2}[\lambda(1+T^{2-2\sigma})X^{1-\sigma}+2(1-\lambda)T^{1-\sigma}]=0$

After the reduction of w_2 between (15) and (16) we find that:

(17) $T^{1-\sigma}[(\mu+1)T^{2-2\sigma}+(1-\mu)][\lambda^3X^{3-2\sigma}-(1-\lambda)^3]-X\lambda(1-\lambda)T^{1-\sigma}[(\mu+3)T^{2-2\sigma}+(3-\mu)][\lambda X^{1-2\sigma}-(1-\lambda)]+$
$X^{1-\sigma}\lambda(1-\lambda)[(\mu+1)T^{4-4\sigma}+4T^{2-2\sigma}+(1-\mu)][\lambda X-(1-\lambda)]-2X^{\sigma}T^{2-2\sigma}[\lambda^3X^{3-4\sigma}-(1-\lambda)^3]=0$

With the substitutions: $T^{1-\sigma}=t\in(0,1)$ and $\alpha=\frac{1-\lambda}{\lambda}>0$, after dividing with λ^3, we find from (17) that:

(18) $t[(\mu+1)t^2+(1-\mu)][X^{3-2\sigma}-\alpha^3]-X\alpha t[(\mu+3)t^2+(3-\mu)][X^{1-2\sigma}-\alpha]+X^{1-\sigma}\alpha[(\mu+1)t^4+4t^2+(1-\mu)][X-\alpha]-2X^\sigma t^2[X^{3-4\sigma}-\alpha^3]=0$

After successive grouping we obtain:

(19) $[X^{1-\sigma}+\alpha t][tX^{1-\sigma}+\alpha]\{[(\mu+1)t^2+(1-\mu)](X-\alpha)-2t(X^{1-\sigma}-\alpha X^\sigma)\}=0$

Because $X^{1-\sigma}+\alpha t>0$ and $tX^{1-\sigma}+\alpha>0$ we have that (19) becomes:

(20) $[(\mu+1)t^2+(1-\mu)](X-\alpha)-2t(X^{1-\sigma}-\alpha X^\sigma)=0$

Let note now:

(21) $\gamma=\frac{(\mu+1)t^2+1-\mu}{4t}$;

$\theta=\alpha^{2\sigma}=\left(\frac{1-\lambda}{\lambda}\right)^{2\sigma}$;

$\varepsilon=2\gamma\alpha^\sigma=\frac{(\mu+1)t^2+1-\mu}{2t}\left(\frac{1-\lambda}{\lambda}\right)^{\sigma}$

$W=\frac{X}{\alpha}=\frac{\lambda}{1-\lambda}X$

We can write (20) like:

(22) $\theta W^{2\sigma-1}+\varepsilon W^\sigma-\varepsilon W^{\sigma-1}-1=0$

We shall call the equation (22) the characteristic equation of the system (3)-(8).

Let now the function: $f:(0,\infty)\to\mathbf{R}$, $f(W)=\theta W^{2\sigma-1}+\varepsilon W^\sigma-\varepsilon W^{\sigma-1}-1$

We have: $\lim_{W\to 0} f(W)=-1$, $f(1)=\theta-1$.

But:

(23) $f'(W)=W^{\sigma-2}[\theta(2\sigma-1)W^\sigma+\varepsilon\sigma W-\varepsilon(\sigma-1)]$

(24) $f''(W)=(\sigma-1)W^{\sigma-3}[2\theta(2\sigma-1)W^\sigma+\varepsilon\sigma W-\varepsilon(\sigma-2)]$

Let $g:(0,\infty)\to\mathbf{R}$, $g(W)=\theta(2\sigma-1)W^\sigma+\varepsilon\sigma W-\varepsilon(\sigma-1)$

Because $g'(W)=\theta\sigma(2\sigma-1)W^{\sigma-1}+\varepsilon\sigma>0$ we have that g has at most one real root.

Because $\lim_{W\to 0} g(W)$ =-ε(σ-1)<0 and g(1)=θ(2σ-1)+εσ-ε(σ-1)=θ(2σ-1)+ε>0 follows $\nu_1 \in (0,1)$ is the real root of g therefore also of f'. We have now: $\lim_{W\to 0} f'(W) < 0$ therefore f'(W)<0 for $W \in (0,\nu_1)$ and f'(W)>0 for $W \in (\nu_1,\infty)$.

Let now h:(0,∞)→**R**, h(W)= $2\theta(2\sigma-1)W^{\sigma}+\varepsilon\sigma W-\varepsilon(\sigma-2)$. We have h'(W)=$2\theta\sigma(2\sigma-1)W^{\sigma-1}+\varepsilon\sigma>0$ therefore h has at most one real root. We have also: $\lim_{W\to 0} h(W)$ =-ε(σ-2) and h(1)=2θ(2σ-1)+εσ-ε(σ-2)=2θ(2σ-1)+2ε>0.

If σ<2 then: $\lim_{W\to 0} h(W)$ >0 therefore h has not real roots. In this case: f"(W)>0.

If σ>2 then $\lim_{W\to 0} h(W)$ <0 and h(1)>0 implies that $\nu_2 \in (0,1)$ is the root of h therefore of f" also. But $\lim_{W\to 0} h(W)$ <0 equivalent with $\lim_{W\to 0} f''(W)$ <0 lead us to the conclusion that: f"(W)<0 for $W \in (0,\nu_2)$ and f"(W)>0 for $W \in (\nu_2,\infty)$. In this case, for the two real roots ν_1 and ν_2, we have from the upper relations:

(25) $\theta(2\sigma-1)\nu_1^{\sigma}+\varepsilon\sigma\nu_1-\varepsilon(\sigma-1)=0$

(26) $2\theta(2\sigma-1)\nu_2^{\sigma}+\varepsilon\sigma\nu_2-\varepsilon(\sigma-2)=0$

We have: $g(\nu_2)=\theta(2\sigma-1)\nu_2^{\sigma}+\varepsilon\sigma\nu_2-\varepsilon(\sigma-1)=-\frac{1}{2}\varepsilon\sigma\nu_2+\frac{1}{2}\varepsilon(\sigma-2)+\varepsilon\sigma\nu_2-\varepsilon(\sigma-1)=\frac{1}{2}\varepsilon\sigma\nu_2-\frac{1}{2}\varepsilon\sigma=\frac{1}{2}\varepsilon\sigma(\nu_2-1)<0$ therefore: $\nu_2<\nu_1$. The root of $f(W)=\theta W^{2\sigma-1}+\varepsilon W^{\sigma}-\varepsilon W^{\sigma-1}-1$ will be greater than ν_1 where the function is strictly increasing and convex. The same thing is obviously in the case σ≤2.

For the determination now of the real root $\overline{W}$ of f, we shall apply the Newton method of approximation for functions of one variable. Because the starting point x_0 for a function f:[a,b]→**R**, who maintains the monotony and the concavity ([2]) is those for which $f(x_0)f''(x_0)>0$ and at us f"(W)>0 for any $W>\nu_1$ we must choose x_0 such that $f(x_0)>0$.

From (21) we have that $\theta=\left(\frac{1-\lambda}{\lambda}\right)^{2\sigma}$ therefore θ>1 if $\lambda\in\left(0,\frac{1}{2}\right)$, θ=1 if $\lambda=\frac{1}{2}$ and θ<1 if $\lambda\in\left(\frac{1}{2},1\right)$. Because f(1)=θ-1, in the case $\lambda\in\left(0,\frac{1}{2}\right)$ we shall choose $x_0=1$ and $\overline{W}<1$. In the case $\lambda=\frac{1}{2}$, from (22) we obtain easy that $\overline{W}=1$. In the case $\lambda\in\left(\frac{1}{2},1\right)$

therefore $\theta<1$ we shall choose x_0 sufficiently large (greater than 1) in order to have convergence of the algorithm and $\overline{W}>1$.

Lemma 1

Let the equation $aW^{2\sigma-1}+bW^{\sigma}-bW^{\sigma-1}-c=0$ where $a,b>0$, $\sigma>1$. In this case, the positive root $\overline{W}$ satisfy: $\overline{W}\leq\left(\dfrac{c+\frac{b}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}{a}\right)^{\frac{1}{2\sigma-1}}$.

Proof

We have $aW^{2\sigma-1}=-bW^{\sigma}+bW^{\sigma-1}+c$

Let the function $f(W)=-bW^{\sigma}+bW^{\sigma-1}+c$.

We have: $f'(W)=-b\sigma W^{\sigma-1}+b(\sigma-1)W^{\sigma-2}=bW^{\sigma-2}[(\sigma-1)-\sigma W]$ therefore $W=\dfrac{\sigma-1}{\sigma}$.

Because $\lim\limits_{W\to 0} f'(W)>0$ we obtain that f increases for $W<\dfrac{\sigma-1}{\sigma}$ and decreases for $W>\dfrac{\sigma-1}{\sigma}$. The function f has a maximum point in $\dfrac{\sigma-1}{\sigma}$ therefore:

$$aW^{2\sigma-1}\leq f\left(\frac{\sigma-1}{\sigma}\right)=b\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(1-\frac{\sigma-1}{\sigma}\right)+c=c+\frac{b}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}.$$

We finally have:

$$\overline{W}\leq\left(\frac{c+\frac{b}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}{a}\right)^{\frac{1}{2\sigma-1}}$$

Lemma 2

Let the equation $\theta W^{2\sigma-1}+\varepsilon W^{\sigma}-\varepsilon W^{\sigma-1}-1=0$ where $\theta,\varepsilon>0$, $\sigma>1$. In this case, the positive root $\overline{W}$ satisfies:

$$\left(\theta+\frac{\varepsilon}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\right)^{-\frac{1}{2\sigma-1}} \le \overline{W} \le \left(\frac{1+\frac{\varepsilon}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}{\theta}\right)^{\frac{1}{2\sigma-1}}$$

Proof

From the lemma 1, for a=θ, b=ε, c=1 we have: $\overline{W} \le \left(\frac{1+\frac{\varepsilon}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}{\theta}\right)^{\frac{1}{2\sigma-1}}$.

Replacing in the equation $\theta W^{2\sigma-1}+\varepsilon W^{\sigma}-\varepsilon W^{\sigma-1}-1=0$ $W=\frac{1}{Y}$ we shall obtain: $Y^{2\sigma-1}+\varepsilon Y^{\sigma}-\varepsilon Y^{\sigma-1}-\theta=0$. From lemma 1, with a=1, b=ε, c=θ we shall obtain: $\frac{1}{\overline{W}} \le \left(\theta+\frac{\varepsilon}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\right)^{\frac{1}{2\sigma-1}}$.

In our case, $\theta=\alpha^{2\sigma}$, $\varepsilon=2\gamma\alpha^{\sigma}$ and $\gamma=\frac{(\mu+1)t^2+1-\mu}{4t}=\frac{(\mu+1)\tau^{2\sigma-2}+1-\mu}{4\tau^{\sigma-1}}$ implies:

$$\alpha^{-\frac{2\sigma}{2\sigma-1}}\left(1+\frac{((\mu+1)\tau^{2\sigma-2}+1-\mu)\alpha^{-\sigma}}{2\sigma}\left(\frac{\sigma-1}{\sigma\tau}\right)^{\sigma-1}\right)^{-\frac{1}{2\sigma-1}} \le \overline{W} \le \alpha^{-\frac{2\sigma}{2\sigma-1}}\left(1+\frac{((\mu+1)\tau^{2\sigma-2}+1-\mu)\alpha^{\sigma}}{2\sigma}\left(\frac{\sigma-1}{\sigma\tau}\right)^{\sigma-1}\right)^{\frac{1}{2\sigma-1}}$$

With the upper relations, we have that for $\alpha=\frac{1-\lambda}{\lambda}$:

$\lambda\in\left(0,\frac{1}{2}\right)$ implies that:

$$\alpha^{-\frac{2\sigma}{2\sigma-1}}\left(1+\frac{((\mu+1)\tau^{2\sigma-2}+1-\mu)\alpha^{-\sigma}}{2\sigma}\left(\frac{\sigma-1}{\sigma\tau}\right)^{\sigma-1}\right)^{-\frac{1}{2\sigma-1}}\leq\overline{W}\leq\min(\alpha^{-\frac{2\sigma}{2\sigma-1}}$$

$$\left(1+\frac{((\mu+1)\tau^{2\sigma-2}+1-\mu)\alpha^{\sigma}}{2\sigma}\left(\frac{\sigma-1}{\sigma\tau}\right)^{\sigma-1}\right)^{\frac{1}{2\sigma-1}},1)$$

$\lambda\in\left(\frac{1}{2},1\right)$ implies :

$$\max(\alpha^{-\frac{2\sigma}{2\sigma-1}}\left(1+\frac{((\mu+1)\tau^{2\sigma-2}+1-\mu)\alpha^{-\sigma}}{2\sigma}\left(\frac{\sigma-1}{\sigma\tau}\right)^{\sigma-1}\right)^{-\frac{1}{2\sigma-1}},1)\leq\overline{W}\leq\alpha^{-\frac{2\sigma}{2\sigma-1}}$$

$$\left(1+\frac{((\mu+1)\tau^{2\sigma-2}+1-\mu)\alpha^{\sigma}}{2\sigma}\left(\frac{\sigma-1}{\sigma\tau}\right)^{\sigma-1}\right)^{\frac{1}{2\sigma-1}}$$

From $\lambda=\frac{1}{2}$ follows: $\overline{W}=1$.

We have now, from the Newton's method:

(27) $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=\frac{2\theta(\sigma-1)x_n^{2\sigma-1}+\varepsilon(\sigma-1)x_n^{\sigma}-\varepsilon(\sigma-2)x_n^{\sigma-1}+1}{\theta(2\sigma-1)x_n^{2\sigma-2}+\varepsilon\sigma x_n^{\sigma-1}-\varepsilon(\sigma-1)x_n^{\sigma-2}}$, $n\geq 0$

where: $\theta=\left(\frac{1-\lambda}{\lambda}\right)^{2\sigma}$ and $\varepsilon=\frac{(\mu+1)t^2+1-\mu}{2t}\left(\frac{1-\lambda}{\lambda}\right)^{\sigma}$, $t=T^{1-\sigma}=\tau^{\sigma-1}$.

The range $(x_n)_{n\geq 0}$ will converges at $\overline{W}$ – the real root of the equation (22).

Using the following formula ([5]):

(28) $\lambda w_1+(1-\lambda)w_2=1$

we finally find that:

(29) $X=\frac{1-\lambda}{\lambda}\overline{W}$

(30) $w_2=\frac{1}{\lambda X+(1-\lambda)}=\frac{1}{(1-\lambda)(\overline{W}+1)}$

(31) $w_1=w_2X=\frac{\overline{W}}{\lambda(\overline{W}+1)}$

(32) $Y_1=\mu\lambda w_1+\frac{1-\mu}{2}=\frac{(1+\mu)\overline{W}+1-\mu}{2(\overline{W}+1)}=\frac{1}{2}+\frac{\mu}{2}\frac{\overline{W}-1}{\overline{W}+1}$

(33) $Y_2=\mu(1-\lambda)w_2+\frac{1-\mu}{2}=\frac{(1-\mu)\overline{W}+1+\mu}{2(\overline{W}+1)}=\frac{1}{2}-\frac{\mu}{2}\frac{\overline{W}-1}{\overline{W}+1}$

(34) $G_1=\frac{\left(\lambda^{\sigma}\overline{W}^{1-\sigma}+(1-\lambda)^{\sigma}\tau^{\sigma-1}\right)^{\frac{1}{1-\sigma}}}{\overline{W}+1}$

(35) $G_2=\frac{\left(\lambda^{\sigma}\tau^{\sigma-1}\overline{W}^{1-\sigma}+(1-\lambda)^{\sigma}\right)^{\frac{1}{1-\sigma}}}{\overline{W}+1}$

(36) $\omega_1=w_1G_1^{-\mu}=\frac{\overline{W}\left(\lambda^{\sigma}\overline{W}^{1-\sigma}+(1-\lambda)^{\sigma}\tau^{\sigma-1}\right)^{-\frac{\mu}{1-\sigma}}}{\lambda\left(\overline{W}+1\right)^{-\mu+1}}$

(37) $\omega_2=w_2G_2^{-\mu}=\frac{\left(\lambda^{\sigma}\tau^{\sigma-1}\overline{W}^{1-\sigma}+(1-\lambda)^{\sigma}\right)^{-\frac{\mu}{1-\sigma}}}{(1-\lambda)\left(\overline{W}+1\right)^{-\mu+1}}$

(38) $\frac{\omega_1}{\omega_2}=\frac{1-\lambda}{\lambda}\tau^{-\mu}\overline{W}\left(1+\frac{(1-\lambda)^{\sigma}(\tau^{2\sigma-2}-1)}{\lambda^{\sigma}\tau^{\sigma-1}\overline{W}^{1-\sigma}+(1-\lambda)^{\sigma}}\right)^{-\frac{\mu}{1-\sigma}}=$

$\alpha\tau^{-\mu}\overline{W}\left(1+\frac{\tau^{2\sigma-2}-1}{\alpha^{-\sigma}\tau^{\sigma-1}\overline{W}^{1-\sigma}+1}\right)^{-\frac{\mu}{1-\sigma}}$

One particular case arises for $\lambda=\frac{1}{2}$ from where $\alpha=1$ therefore $\overline{W}=1$. We have now:

(39) $w_2=1$

(40) $w_1=1$

(41) $Y_1=\frac{1}{2}$

(42) $Y_2=\frac{1}{2}$

(43) $G_1=\left(\frac{1+\tau^{\sigma-1}}{2}\right)^{\frac{1}{1-\sigma}}$

(44) $G_2=\left(\frac{1+\tau^{\sigma-1}}{2}\right)^{\frac{1}{1-\sigma}}$

(45) $\omega_1=w_1G_1^{-\mu}=\left(\frac{1+\tau^{\sigma-1}}{2}\right)^{-\frac{\mu}{1-\sigma}}$

(46) $\omega_2=w_2G_2^{-\mu}=\left(\frac{1+\tau^{\sigma-1}}{2}\right)^{-\frac{\mu}{1-\sigma}}$

(47) $\frac{\omega_1}{\omega_2}=1$.

3. Conclusions

With the method presented in the previous section, we have determined all the cases for $\mu\in[1/10,9/10]$ with step=1/10, $\sigma\in\{2,3,4,5\}$ and $\tau\in[10/100,99/100]$ with step=1/100.

We can easily see that, related to the variation of λ:

- $\frac{\omega_1}{\omega_2}$ is an increasing function if:

μ	**σ**	**τ**	**μ**	**σ**	**τ**	**μ**	**σ**	**τ**
0.1	**2**	**[0.55,1)**	**0.4**	**2**	**[0.1,1)**	**0.7**	**2**	**[0.1,1)**
0.1	**3**	**[0.78,1)**	**0.4**	**3**	**[0.33,1)**	**0.7**	**3**	**[0.1,1)**
0.1	**4**	**[0.86,1)**	**0.4**	**4**	**[0.51,1)**	**0.7**	**4**	**[0.19,1)**
0.1	**5**	**[0.9,1)**	**0.4**	**5**	**[0.62,1)**	**0.7**	**5**	**[0.33,1)**
0.2	**2**	**[0.29,1)**	**0.5**	**2**	**[0.1,1)**	**0.8**	**2**	**[0.1,1)**
0.2	**3**	**[0.6,1)**	**0.5**	**3**	**[0.22,1)**	**0.8**	**3**	**[0.1,1)**
0.2	**4**	**[0.73,1)**	**0.5**	**4**	**[0.41,1)**	**0.8**	**4**	**[0.1,1)**
0.2	**5**	**[0.8,1)**	**0.5**	**5**	**[0.53,1)**	**0.8**	**5**	**[0.1,1)**
0.3	**2**	**[0.14,1)**	**0.6**	**2**	**[0,1,1)**	**0.9**	**2**	**[0.1,1)**
0.3	**3**	**[0.46,1)**	**0.6**	**3**	**[0.12,1)**	**0.9**	**3**	**[0.1,1)**
0.3	**4**	**[0.62,1)**	**0.6**	**4**	**[0.31,1)**	**0.9**	**4**	**[0.1,1)**
0.3	**5**	**[0.71,1)**	**0.6**	**5**	**[0.44,1)**	**0.9**	**5**	**[0.1,1)**

Analyzing these, we can easily see that for a large share of expenditure which is received by the manufacturers **-** μ, the ratio of the real wages of workers living in the corresponding regions increases in reference with the share of manufacturing labor force in the first region, even if it is a good or poor distribution of manufactured goods or a smaller elasticity of demand.

If μ has a little value, that is the character of consumption in the first region is preponderant agricultural, a bigger elasticity of demand and a good policy of transportation will contribute to an increasing of the ratio of the real wages.

If we compute the total variation of the function we can remark that it essential depends from μ, that is how much the manufacturing industry has a dominant role in the region, an increase of the manufacturing labor force in the first region will contribute to a bigger increasing of $\frac{\omega_1}{\omega_2}$.

- $\frac{\omega_1}{\omega_2}$ is a decreasing function if:

μ	**σ**	**τ**
0.1	**2**	**(0,0.53]**
0.1	**3**	**(0,0.77]**
0.1	**4**	**(0,0.85]**
0.1	**5**	**(0,0.89]**
0.2	**2**	**(0,0.2]**
0.2	**3**	**(0,0.56]**
0.2	**4**	**(0,0.7]**
0.2	**5**	**(0,0.77]**
0.3	**3**	**(0,0.27]**
0.3	**4**	**(0,0.49]**
0.3	**5**	**(0,0.62]**
0.4	**4**	**(0,0.14]**
0.4	**5**	**(0,0.22]**

For a region preponderant agricultural related to the consumption and a poor distribution of goods or in the case of a big elasticity of demand, the ratio always will decrease.

Again, the total variation of the function shows us that a little decreasing of $\frac{\omega_1}{\omega_2}$ it is possible only if we have a good distribution of goods to the other region.

- $\frac{\omega_1}{\omega_2}$ has an alternate character (first increases, after decreases and finally increase) if:

μ	σ	τ	μ	σ	τ
0.1	**2**	**[0.54,0.54]**	**0.4**	**4**	**[0.15,0.5]**
0.2	**2**	**[0.21,0.28]**	**0.4**	**5**	**[0.23,0.61]**
0.2	**3**	**[0.57,0.59]**	**0.5**	**3**	**[0.1,0.21]**
0.2	**4**	**[0.71,0.72]**	**0.5**	**4**	**[0.1,0.4]**
0.2	**5**	**[0.78,0.79]**	**0.5**	**5**	**[0.1,0.52]**
0.3	**2**	**[0.1,0.13]**	**0.6**	**3**	**[0.1,0.11]**
0.3	**3**	**[0.28,0.45]**	**0.6**	**4**	**[0.1,0.3]**
0.3	**4**	**[0.5,0.61]**	**0.6**	**5**	**[0.1,0.43]**
0.3	**5**	**[0.63,0.7]**	**0.7**	**4**	**[0.1,0.18]**
0.4	**3**	**[0.1,0.32]**	**0.7**	**5**	**[0.1,0.32]**

If the variation is pendulous we can see that the total variation is not so big. It has large values for big values of σ that is for an important elasticity of demand.

Bibliography

Ioan, C.A. (2008). *Mathematics I*. Bucharest: Sinteze.

Ioan, C.A. (2008). *Mathematics II*. Bucharest: Sinteze.

Krugman, P. (1991). Increasing Returns and Economic Geography. *Journal of Political Economy*, 99, pp. 483-499.

Krugman, P. (1998). What's New About The New Economic Geography? *Oxford Review of Economic Policy*, Vol.14, No. 2, pp. 7-17.

Mossay, P. (2006). The Core-Periphery Model: Existence and Uniqueness of Short-run Equilibrium, Instituto Valenciano de Investigaciones Economicas, S.A. *Working Papers, AD Series*, Vol. 59, No. 3, pp. 383-393.